Mathematics

Mathematics
1 Using and applying mathematics

Level 1 Pupils use mathematics as an integral part of classroom activities. They represent their work with objects or pictures and discuss it. They recognise and use a simple pattern or relationship.

Level 2 Pupils select the mathematics they use in some classroom activities. They discuss their work using mathematical language and are beginning to represent it using symbols and simple diagrams. They explain why an answer is correct.

Level 3 Pupils try different approaches and find ways of overcoming difficulties that arise when they are solving problems. They are beginning to organise their work and check results. Pupils discuss their mathematical work and are beginning to explain their thinking. They use and interpret mathematical symbols and diagrams. Pupils show that they understand a general statement by finding particular examples that match it.

Level 4 Pupils develop their own strategies for solving problems and use these strategies both in working within mathematics and in applying mathematics to practical contexts. When solving problems, with or without ICT, they check their results are reasonable by considering the context. They look for patterns and relationships, presenting information and results in a clear and organised way, using ICT appropriately. They search for a solution by trying out ideas of their own.

Level 5 In order to explore mathematical situations, carry out tasks or tackle problems, pupils identify the mathematical aspects and obtain necessary information. They calculate accurately, using ICT where appropriate. They check their working and results, considering whether these are sensible. They show understanding of situations by describing them mathematically using symbols, words and diagrams. They draw simple conclusions of their own and explain their reasoning.

Level 6 Pupils carry out substantial tasks and solve quite complex problems by independently and systematically breaking them down into smaller, more manageable tasks. They interpret, discuss and synthesise information presented in a variety of mathematical forms, relating findings to the original context. Their written and spoken language explains and informs their use of diagrams. They begin to give mathematical justifications, making connections between the current situation and situations they have encountered before.

Level 7 Starting from problems or contexts that have been presented to them, pupils explore the effects of varying values and look for invariance in models and representations, working with and without ICT. They progressively refine or extend the mathematics used, giving reasons for their choice of mathematical presentation and explaining features they have selected. They justify their generalisations, arguments or solutions, looking for equivalence to different problems with similar structures. They appreciate the difference between mathematical explanation and experimental evidence.

Level 8 Pupils develop and follow alternative approaches. They compare and evaluate representations of a situation, introducing and using a range of mathematical techniques. They reflect on their own lines of enquiry when exploring mathematical tasks. They communicate mathematical or statistical meaning to different audiences through precise and consistent use of symbols that is sustained throughout the work. They examine generalisations or solutions reached in an activity and make further progress in the activity as a result. They comment constructively on the reasoning and logic, the process employed and the results obtained.

Exceptional Performance Pupils critically examine the strategies adopted when investigating within mathematics itself or when using mathematics to analyse tasks. They explain why different strategies were used, considering the elegance and efficiency of alternative lines of enquiry or procedures. They apply the mathematics they know in a wide range of familiar and unfamiliar contexts. They use mathematical language and symbols effectively in presenting a convincing, reasoned argument. Their reports include mathematical justifications, distinguishing between evidence and proof and explaining their solutions to problems involving a number of features or variables.

Mathematics
2 Number and algebra

Level 1 Pupils count, order, combine, increase and decrease quantities when solving problems in practical contexts. They read and write the numbers involved.

Level 2 Pupils count sets of objects reliably, and use mental recall of addition and subtraction facts to 10. They begin to understand the place value of each digit in a number and use this to order numbers up to 100. They choose the appropriate operation when solving addition and subtraction problems. They use the knowledge that subtraction is the inverse of addition. They use mental calculation strategies to solve number problems involving money and measures. They recognise sequences of numbers, including odd and even numbers.

Level 3 Pupils show understanding of place value in numbers up to 1000 and use this to make approximations. They begin to use decimal notation, in the context of measures and money, and to recognise negative numbers in practical contexts such as temperature. Pupils use mental recall of addition and subtraction facts to 20 in solving problems involving larger numbers. They add and subtract numbers with two digits mentally and numbers with three digits using written methods. They use mental recall of the 2, 3, 4, 5 and 10 multiplication tables and derive the associated division facts. They solve whole-number problems involving multiplication or division including those that give rise to remainders. They use simple fractions that are several parts of a whole and recognise when two simple fractions are equivalent.

Level 4 Pupils use their understanding of place value to mentally multiply and divide whole numbers by 10 or 100. When solving number problems, they use a range of mental methods of computation with the four operations, including mental recall of multiplication facts up to 10 x 10 and quick derivation of corresponding division facts. They select efficient strategies for addition, subtraction, multiplication and division. They recognise approximate proportions of a whole and use simple fractions and percentages to describe these. They begin to use simple formulae expressed in words.

Level 5 Pupils use their understanding of place value to multiply and divide whole numbers and decimals. They order, add and subtract negative numbers in context. They use all four operations with decimals to two places. They solve simple problems involving ratio and direct proportion. They calculate fractional or percentage parts of quantities and measurements, using a calculator where appropriate. They construct, express in symbolic form and use simple formulae involving one or two operations. They use brackets appropriately. They use and interpret coordinates in all four quadrants.

Level 6 Pupils order and approximate decimals when solving numerical problems and equations, using trial and improvement methods. They evaluate one number as a fraction or percentage of another. They understand and use the equivalences between fractions, decimals and percentages, and calculate using ratios in appropriate situations. They add and subtract fractions by writing them with a common denominator. They find and describe in words the rule for the next term or nth term of a sequence where the rule is linear. They formulate and solve linear equations with whole-number coefficients. They represent mappings expressed algebraically, and use Cartesian coordinates for graphical representation interpreting general features.

Level 7 When making estimates, pupils round to one significant figure and multiply and divide mentally. They understand the effects of multiplying and dividing by numbers between 0 and 1. They solve numerical problems involving multiplication and division with numbers of any size, using a calculator efficiently and appropriately. They understand and use proportional changes, calculating the result of any proportional change using only multiplicative methods. They find and describe in symbols the next term or nth term of a sequence where the rule is quadratic. They use algebraic and graphical methods to solve simultaneous linear equations in two variables.

Level 8 Pupils solve problems that involve calculating with powers, roots and numbers expressed in standard form. They choose to use fractions or percentages to solve problems involving repeated proportional changes or the calculation of the original quantity given the result of a proportional change. They evaluate algebraic formulae or calculate one variable, given the others, substituting fractions, decimals and negative numbers. They manipulate algebraic formulae, equations and expressions, finding common factors and multiplying two linear expressions. They solve inequalities in two variables. They sketch and interpret graphs of linear, quadratic, cubic and reciprocal functions, and graphs that model real situations.

Exceptional Performance Pupils understand and use rational and irrational numbers. They determine the bounds of intervals. They understand and use direct and inverse proportion. In simplifying algebraic expressions, they use rules of indices for negative and fractional values. In finding formulae that approximately connect data, they express general laws in symbolic form. They solve simultaneous equations in two variables where one equation is linear and the other is quadratic. They solve problems using intersections and gradients of graphs.

Mathematics
3 Shape, space and measures

Level 1 When working with 2-D and 3-D shapes, pupils use mathematical language to describe properties and positions. They measure and order objects using direct comparison, and order events.

Level 2 Pupils use mathematical names for common 3-D and 2-D shapes and describe their properties, including numbers of faces, edges and vertices. They distinguish between straight and turning movements, recognise angle as a measurement of turn, and right angles in turns. They begin to use everyday non-standard and standard units to measure length and mass.

Level 3 Pupils classify 3-D and 2-D shapes in various ways using mathematical properties such as reflective symmetry for 2-D shapes. They use non-standard units, standard metric units of length including finding perimeters, capacity and mass, and standard units of time, in a range of contexts.

Level 4 Pupils use and make geometric 2-D and 3-D patterns, scale drawings and models in practical contexts. They reflect simple shapes in a mirror line. They choose and use appropriate units and tools, interpreting, with appropriate accuracy, numbers on a range of measuring instruments. They find areas of simple shapes.

Level 5 When constructing models and drawing or using shapes, pupils measure and draw angles to the nearest degree and use language associated with angles. They know the angle sum of a triangle and that of angles at a point. They identify all the symmetries of 2-D shapes. They convert one metric unit to another. They make sensible estimates of a range of measures in relation to everyday situations. They understand and use the formula for the area of a rectangle.

Level 6 Pupils recognise and use common 2-D representations of 3-D objects. They know and use the properties of quadrilaterals. They solve problems using angle and symmetry, properties of polygons and angle properties of intersecting and parallel lines, and explain these properties. They devise instructions for a computer to generate and transform shapes and paths. They understand and use appropriate formulae for finding circumferences and areas of circles, areas of plane rectilinear figures and volumes of cuboids when solving problems.

Level 7 Pupils understand and apply Pythagoras' theorem when solving problems in two dimensions. They calculate lengths, areas and volumes in plane shapes and right prisms. They enlarge shapes by a fractional scale factor, and appreciate the similarity of the resulting shapes. They determine the locus of an object moving according to a rule. They appreciate the imprecision of measurement and recognise that a measurement given to the nearest whole number may be inaccurate by up to one half in either direction. They understand and use compound measures, such as speed.

Level 8 Pupils understand and use congruence and mathematical similarity. They use sine, cosine and tangent in right-angled triangles when solving problems in two dimensions.

Exceptional Performance Pupils sketch the graphs of sine, cosine and tangent functions for any angle, and generate and interpret graphs based on these functions. They use sine, cosine and tangent of angles of any size, and Pythagoras' theorem when solving problems in two and three dimensions. They construct formal geometric proofs. They calculate lengths of circular arcs and areas of sectors, and calculate the surface area of cylinders and volumes of cones and spheres. They appreciate the continuous nature of scales that are used to make measurements.

Mathematics
4 Statistics

Level 1 Pupils sort objects and classify them, demonstrating the criterion they have used. They collect data to answer questions.

Level 2 Pupils sort objects and classify them using more than one criterion. When they have gathered information to answer a question or explore a situation, pupils record results in simple lists, tables, diagrams and block graphs, in order to communicate their findings.

Level 3 Pupils extract and interpret information presented in simple tables and lists. They construct charts and diagrams to communicate information they have gathered for a purpose, and they interpret information presented to them in this form.

Level 4 Pupils generate and answer questions that require the collection of discrete data which they record using a frequency table. They understand and use an average and range to describe sets of data. Using technology where appropriate: they group data in equal class intervals if necessary, represent collected data in frequency diagrams and interpret such diagrams. They construct and interpret simple line graphs.

Level 5 Pupils understand and use the mean of discrete data. They compare two simple distributions using the range and one of the mode, median or mean. They interpret graphs and diagrams, including pie charts, and draw conclusions. They understand and use the probability scale from 0 to 1. They find and justify probabilities and approximations to these by selecting and using methods based on equally likely outcomes and experimental evidence, as appropriate. They understand that different outcomes may result from repeating an experiment.

Level 6 Pupils collect and record continuous data, choosing appropriate equal class intervals over a sensible range to create frequency tables. They construct and interpret frequency diagrams. They construct pie charts. They draw conclusions from scatter diagrams, and have a basic understanding of correlation. When dealing with a combination of two experiments, they identify all the outcomes. When solving problems, they use their knowledge that the total probability of all the mutually exclusive outcomes of an experiment is 1.

Level 7 Pupils specify hypotheses and test them by designing and using appropriate methods that take account of variability or bias. They determine the modal class and estimate the mean, median and range of sets of grouped data, selecting the statistic most appropriate to their line of enquiry. They use measures of average and range, with associated frequency polygons, as appropriate, to compare distributions and make inferences. They understand relative frequency as an estimate of probability and use this to compare outcomes of experiments.

Level 8 Pupils interpret and construct cumulative frequency tables and diagrams. They estimate the median and interquartile range and use these to compare distributions and make inferences. They understand how to calculate the probability of a compound event and use this in solving problems.

Exceptional Performance Pupils interpret and construct histograms. They understand how different methods of sampling and different sample sizes may affect the reliability of conclusions drawn. They select and justify a sample and method to investigate a population. They recognise when and how to work with probabilities associated with independent, mutually exclusive events.